

## Exam I, MTH 101, Spring 2015

Ayman Badawi

53/55

## QUESTION 1.

For a certain product, let  $p(x) = 100 - 4x$  be the selling price per item in dirhams, where  $x$  is the number of items in hundreds. Let  $C(x) = 200 + 10x$  be the total cost function in hundreds of dirhams. Assume that  $0 \leq x \leq 25$ .

(i) (2 points) Find the Revenue function  $R(x)$  and the Profit function  $P(x)$ .

$$R(x) = x p(x) = -4x^2 + 100x$$

$$\begin{aligned} P(x) &= R(x) - C(x) = (-4x^2 + 100x) - (200 + 10x) \\ &= -4x^2 + 100x - 10x - 200 \\ &= -4x^2 + 90x - 200 \end{aligned}$$

(ii) (2 points) How many items should be sold in order to maximize the profit?

$$\text{max items} = -\frac{b}{2a} \text{ of } P(x)$$

$$\frac{-b}{2a} = \frac{-90}{2(-4)} = 11.25 \text{ hundreds}$$

(iii) (2 points) Calculate the break even points.

$$P(x) = 0$$

$$-4x^2 + 90x - 200 = 0$$

$$x = 20 \quad x = 2.5 \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} \text{ hundreds}$$

are the break even points.

$$x = \frac{-90 \pm \sqrt{90^2 - 4(-4)(-200)}}{2(-4)}$$

$$x = 20$$

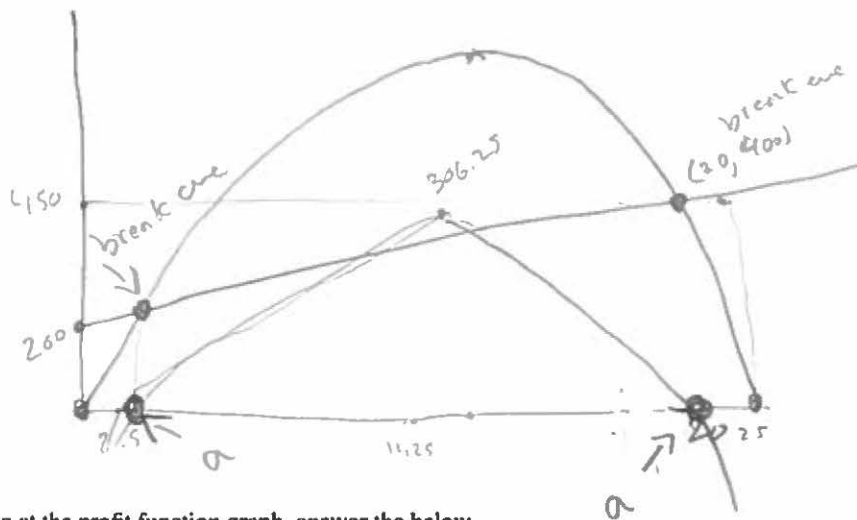
$$x = 2.5$$

(iv) (2 points) What is the selling price per item that produces the maximum profit?

$$p(11.25) = 100 - 4(11.25) = 55 \text{ hundreds}$$

(v) (5 points) Roughly, Sketch the graphs of  $R(x)$ ,  $P(x)$  and  $C(x)$ . (note that in all these functions  $0 \leq x \leq 25$ )

Nader



$$R(x) = -4x^2 + 100x$$

$$-\frac{b}{2a} = \frac{-100}{2(-4)} = 12.5$$

$$C(x) = 200 + 10x$$

$$C(0) = 200$$

$$C(25) = 450$$

By starting at the profit function graph, answer the below

- Locate the break even points on the profit graph.
- For what values of  $x$  do we have a loss?

$$x < 2.5 \text{ or } x > 20$$

- For what values of  $x$  do we make a profit?

$$2.5 < x < 20$$

### QUESTION 2.

(5 points) A phone-company has the following rates on international calls.

10 drs for the first 8 minutes or less

1 dhs per minute for the next 12 minutes

0.8 dhs per minute for all over 20 minutes

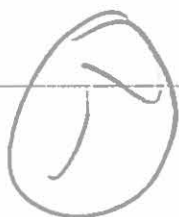
Write a piecewise function for the charge  $C(x)$  (in dirhams) for a customer who uses  $x$  minutes. How much will you pay if you made a 25-minutes call?

$$C(x) = \begin{cases} 10 & x \leq 8 \\ 10 + 1(x-8) & 8 < x \leq 12 \\ 10 + 1(12) + 0.8(x-12) & x > 12 \end{cases}$$

$$10 + 12 + 0.8(25-12) = 32.4 \text{ DHS}$$

$$\downarrow$$

$$20 \quad 26$$



**QUESTION 3. (3 points)** At Islamic-Sharjah Bank, you can put your money in an account such that the APY is 4.86%. What annual interest rate, compounded *monthly*, has an APY of 4.86%?

$$APY = 4.86\% = 0.0486$$

$$0.0486 = \left(1 + \frac{r}{12}\right)^{12} - 1$$

$$\sqrt[12]{1.0486} - 1 = \frac{r}{12} \quad r = 0.0475$$

$$(1.033 - 1) \times 12 = r \quad r = 4.75\%$$

**QUESTION 4. (9 points)** Suppose if we have decided to assume that every month you are making a deposit of 1000 Dhs into an account that gives annual interest rate 6%, compounded monthly. How much interest is earned during the 3<sup>rd</sup> year?

$$FV_2 = 1000 \left( \frac{\left(1 + \frac{0.06}{12}\right)^{24} - 1}{\frac{0.06}{12}} \right) = 25431.95$$

$$FV_3 = 1000 \left( \frac{\left(1 + \frac{0.06}{12}\right)^{36} - 1}{\frac{0.06}{12}} \right) = 39336.10$$

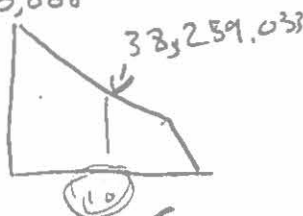
$$I = \text{total pmt in one year} - (FV_3 - FV_2)$$

$$= 1000 \times 12 - (39336.10 - 25431.95)$$

$$I = 1904.15$$

**QUESTION 5. (12 points)** Imagine the following case: You took a loan of 80,000 dirhams to be paid by making monthly payments for 15 years at annual interest rate of 8.4% compounded monthly. After 10 years the bank reduces its annual interest to 6% and you agreed to repay the remaining amount of loan over 5 years at the new annual rate of 6% compounded monthly. What is your new monthly payment?

$$Pmt = \frac{80,000 \cdot \frac{0.084}{12}}{1 - \left(1 + \frac{0.084}{12}\right)^{-180}}$$



$$Pmt = 783.10 \text{ at the beginning}$$

$$PV = 783.10 \left( \frac{1 - \left(1 + \frac{0.084}{12}\right)^{-60}}{\frac{0.084}{12}} \right) = 38,259.03$$

$$Pmt = \frac{38,259.03 \times \frac{0.06}{12}}{1 - \left(1 + \frac{0.06}{12}\right)^{-60}} = 739.65$$

**QUESTION 6. (4 points)** Imagine we have the following case: there is a car of type BAD. The price of BAD-car has been increasing at a rate of 8% compounded quarterly over the last 3 years. If today the price of a BAD-car is 140,000 dirhams. What would have been the price of the BAD-car 3 years ago?

$$P = \frac{140000}{(1 + \frac{0.08}{4})^{12}} = 129338.35$$

~~12~~      1

**QUESTION 7.** Imagine this case: My son Nadeem decided to invest his money for 9 months, so he deposited 500 dirhams in Sharjah-Islamic Bank where an annual simple interest 10% is given. After 3 months, he gave up his bank account to his sister Raneem and Raneem paid him 510 Dhs.

(i) (3 points) What annual simple interest rate will Nadeem receive for this investment?

$$\frac{\frac{A}{P} - 1}{t} = r = \frac{\frac{510}{500} - 1}{\frac{3}{12}} = 0.08 \quad 8\%$$

(ii) (3 points) What annual simple interest rate will Raneem receive for her investment?

before:

$$A = 500 \left(1 + 0.1 \times \frac{9}{12}\right) = 537.5$$

After:

$$537.5 = 510 \left(1 + r \times \frac{6}{12}\right)$$

$$\frac{\frac{537.5}{510} - 1}{\frac{6}{12}} = r = 0.1078$$

$$= 10.78\%$$

Good

**QUESTION 8. (3 points)** You want to invest money. You have the choice between the following two different banks:

**Bank A** offers 8% compounded semi annually.

**Bank B** offers 7.6% compounded quarterly.

Which one do you choose and why?

$$\checkmark APY_1 = \left(1 + \frac{0.08}{2}\right)^2 - 1 = 0.0816 = 8.16\%$$

$$APY_2 = \left(1 + \frac{0.076}{4}\right)^4 - 1 = 0.07819 = 7.819\%$$

APY<sub>1</sub>

#### Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.  
E-mail: [abadawi@aus.edu](mailto:abadawi@aus.edu), [www.ayman-badawi.com](http://www.ayman-badawi.com)